

Inflation with large gravitational waves.

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It is well known that in manifestly Lorentz invariant theories with nontrivial kinetic terms, perturbations around some classical backgrounds can travel faster than light. These exotic "supersonic" models may have interesting consequences for cosmology and astrophysics. In particular, one can show¹ that in such theories the contribution of the gravitational waves to the CMB fluctuations can be significantly larger than that in standard inflationary models. This increase of the tensor-to-scalar perturbation ratio leads to a larger B-component of the CMB polarization, thus making the prospects for future detection much more promising. Interestingly, the spectral index of scalar perturbations and mass of the scalar field considered in the model are practically indistinguishable from the standard case. Whereas the energy scale of inflation and hence the reheating temperature can be much higher compared to a simple chaotic inflation.

1 Introduction

One of the main consequences of inflation is the generation of primordial cosmological perturbations² and the production of long wavelength gravitational waves (tensor perturbations)³. The predicted slightly red-tilted spectrum of the scalar perturbations is at present in excellent agreement with the measurements of the CMB fluctuations⁴. The observation of primordial gravitational waves together with the detection of a small deviation of the spectrum from flat would give us further strong confirmation of inflationary paradigm. The detection of primordial gravitational waves is not easy, but they can be seen indirectly in the B-mode of the CMB polarization (see, for example,⁵). In standard slow-roll inflationary scenarios⁶ the amplitude of the tensor perturbations can, in principle, be large enough to be observed. However, it is only on the border of detectability in future experiments.

There are a lot of inflationary scenarios where the tensor component produced during inflation is much less than that in the chaotic inflation. In particular, in models such as new inflation⁷ and hybrid inflation⁸, tensor perturbations are typically small⁵. Moreover, in the curvaton scenario⁹ and k-inflation¹⁰, they can be suppressed completely.

A natural question is whether the gravitational waves can be significantly enhanced compared to standard scenarios. Recently it was argued that the contribution of tensor perturbations to the CMB anisotropy can be much greater than expected^{11,12}. However, it was found in¹³ that in the models considered in^{11,12} one cannot avoid the production of too large scalar perturbations and therefore they are in contradiction with observations.

In the paper¹ we introduced a class of inflationary models where the B-mode of polarization can exceed that predicted by simple chaotic inflation. These models resemble both k-inflation¹⁰ and chaotic inflation⁶. Inflation occurs due to the potential term in the Lagrangian, and the kinetic term has a nontrivial structure responsible for the large sound speed of perturbations.

In this talk I will review the model from ¹.

2 Basic equations and main idea

The generic action describing a scalar field interacting with the gravitational field is

$$S = S_g + S_\varphi = \int d^4x \sqrt{-g} \left[-\frac{R}{16\pi} + p(\phi, X) \right], \quad (1)$$

where R is the Ricci scalar and $p(\phi, X)$ is a function of the scalar field ϕ and its first derivatives $X = \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$. We use Planck units, where $G = \hbar = c = 1$. In the case of the usual scalar field the X -dependence of p is trivial, namely, $p = X - V(\phi)$, while k-inflation and k-essence ^{10,14,5} are based on the non-trivial dependence of p on X . For $X > 0$, variation of the action (1) with respect to the metric gives the energy momentum tensor for the scalar field in the form of an “hydrodynamical fluid”:

$$T^\mu_\nu = (\varepsilon + p)u^\mu u_\nu - p\delta^\mu_\nu. \quad (2)$$

Here the Lagrangian $p(\phi, X)$ plays the role of pressure, the “four-velocity” is $u_\mu = \nabla_\mu \phi / \sqrt{2X}$, and the energy density is given by $\varepsilon = 2Xp_{,X} - p$ where $p_{,X} = \partial p / \partial X$. Let us consider a spatially flat Friedmann universe with small perturbations:

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t) [(1 - 2\Phi) \delta_{ik} + h_{ik}] dx^i dx^k, \quad (3)$$

where Φ is the gravitational potential characterizing scalar metric perturbations and h_{ik} is a traceless, transverse perturbations describing the gravitational waves. The minimal set of equations for the evolution of the scale factor $a(t)$ and the scalar field $\phi(t)$ is given by

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} \varepsilon \quad \text{and} \quad \ddot{\phi} + 3c_S^2 H \dot{\phi} + \frac{\varepsilon_{,\phi}}{\varepsilon_{,X}} = 0, \quad (4)$$

where the dot denotes the derivative with respect to time t and the “speed of sound” is

$$c_S^2 \equiv \frac{p_{,X}}{\varepsilon_{,X}} = \left[1 + 2X \frac{p_{,XX}}{p_{,X}} \right]^{-1}. \quad (5)$$

One can show that c_S is in fact the speed of propagation of the cosmological perturbations ^{15,5}. The stability condition with respect to the high frequency cosmological perturbations requires $c_S^2 > 0$. For simplicity let us consider theories with Lagrangians of the form $p = K(X) - V(\phi)$. From the equations of motion (4) it is clear that, if the slow-roll conditions

$$XK_{,X} \ll V, \quad \text{and} \quad K \ll V, \quad \left| \ddot{\phi} \right| \ll \frac{V_{,\phi}}{\varepsilon_{,X}} \quad (6)$$

are satisfied for at least 75 e-folds then we have a successful slow-roll inflation due to the potential V . In contrast to ordinary slow-roll inflation one can arrange here practically any speed of sound c_S^2 by taking an appropriate kinetic term $K(X)$ ¹. The crucial point is that the amplitude of the final scalar perturbations (during the postinflationary, radiation-dominated epoch) and the ratio of tensor to scalar amplitudes on supercurvature scales are given by (see, ⁵):

$$\delta_\Phi^2 \simeq \frac{64}{81} \left(\frac{\varepsilon}{c_S (1 + p/\varepsilon)} \right)_{c_S k \simeq Ha}, \quad \frac{\delta_h^2}{\delta_\Phi^2} \simeq 27 \left(c_S \left(1 + \frac{p}{\varepsilon} \right) \right)_{k \simeq Ha}. \quad (7)$$

Here it is worthwhile reminding that all physical quantities on the right hand side of Eqs. (7) have to be calculated during inflation at the moment when perturbations with wave number k cross corresponding Horizon: $c_S k \simeq Ha$ for δ_Φ and $k \simeq Ha$ for δ_h respectively. The amplitude of the scalar perturbations δ_Φ is a free parameter of the theory which is taken to fit the observations. Therefore, it follows from (7) that the tensor-to-scalar ratio can be arbitrarily enhanced in such models.

3 “Simple” model

As a concrete example let us consider a simple model with Lagrangian

$$p(\phi, X) = \alpha^2 \left[\sqrt{1 + \frac{2X}{\alpha^2}} - 1 \right] - \frac{1}{2} m^2 \phi^2, \quad (8)$$

where constant α is a free parameter. For $2X \ll \alpha^2$ one recovers the Lagrangian for the usual free scalar field. The function p is a monotonically growing concave function of X , therefore the system is ghost-free. The effective speed of sound, $c_s^2 = 1 + 2X/\alpha^2$, is larger than the speed of light, approaching it as $X \rightarrow 0$. In the slow-roll regime and for p given in (8), equations (4) reduce to

$$H \simeq \sqrt{\frac{4\pi}{3}} m \phi, \quad 3p_{,X} H \dot{\phi} + m^2 \phi \simeq 0. \quad (9)$$

For $12\pi\alpha^2 > m^2$ there exists a slow-roll solution:

$$\dot{\phi} \simeq -\frac{mc_\star}{\sqrt{12\pi}}, \quad \text{where} \quad c_\star = \left(1 - \frac{m^2}{12\pi\alpha^2} \right)^{-1/2}, \quad (10)$$

is the sound speed during inflation. The sound speed is constant and can be arbitrarily large, if we take $12\pi\alpha^2 \rightarrow m^2$. The pressure and energy density during the slow-roll regime are given by

$$p \simeq m^2 \left(\frac{1}{12\pi} \frac{c_\star^2}{1 + c_\star} - \frac{\phi^2}{2} \right), \quad \varepsilon \simeq m^2 \left(\frac{1}{12\pi} \frac{c_\star}{1 + c_\star} + \frac{\phi^2}{2} \right), \quad (11)$$

respectively. And for the scale factor we have

$$a(\phi) \simeq a_f \exp \left(\frac{2\pi}{c_\star} (\phi_f^2 - \phi^2) \right), \quad (12)$$

where we have introduced subscript f for the quantities at the end of inflation. The inflation is over when $(\varepsilon + p)/\varepsilon \simeq c_\star/(6\pi\phi^2)$ becomes of order unity, that is, at $\phi \sim \phi_f = \sqrt{c_\star/6\pi}$. After that the field ϕ begins to oscillate and decays. Given a number of e-folds before the end of inflation N , we find that at this time $2\pi\phi^2/c_\star \simeq N$, and, hence, $(\varepsilon + p)/\varepsilon \simeq 1/3N$ does not depend on c_\star . Thus, for a given scale, which crosses the Hubble scale N e-folds before the end of inflation, the tensor-to-scalar ratio is $\delta_h^2/\delta_\phi^2 \simeq 27c_\star(1 + p/\varepsilon) \simeq 9c_\star/N$. It is clear that by choosing α close to the critical value $m/\sqrt{12\pi}$ we can have a very large c_\star and consequently enhance this ratio almost arbitrarily. Finally one can estimate the mass which is needed in order to reproduce the observed $\delta_\phi \sim 10^{-5}$. Combining estimations made above we obtain $m \simeq 3\sqrt{3\pi}\delta_\phi/4N$ or for $N \sim 60$, $m \sim 3 \cdot 10^{-7}$ similarly to the usual chaotic inflation. The spectral index of scalar perturbations reads⁵: $n_s - 1 \simeq -3(1 + p/\varepsilon) - H^{-1}d(\ln(1 + p/\varepsilon))/dt \simeq -2/N$ this is exactly the same tilt as for the usual chaotic inflation.

4 Conclusions

We have shown above that in theories where the Lagrangian is a nontrivial, nonlinear function of the kinetic term, the scale of inflation can be pushed to a very high energies without coming into conflict with observations. As a result, the amount of produced gravitational waves can be much larger than is usually expected. If such a situation were realized in nature then the prospects for the future detection of the B-mode of CMB polarization are greatly improved. Of course, the theories where this happens are very unusual. For example, the Cauchy problem is well-posed not for all initial data^{16,17}. Moreover, the horizons lose their universality^{18,19}. Therefore, future observations of the CMB fluctuations are extremely important since they will not only restrict the number of possible candidates for the inflaton but also shed light on the problem of the “superluminal” propagation.

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